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SHORT COMMUNICATIONS

Optimization of Passive Orbit with the Use of Gravity Maneuver

J. J. Smulsky

Institute of the Earth's Cryosphere, Siberian Branch of Russian Academy of Sciences
E-mail: JSmulsky@mail.ru

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Abstract—A high-precision method of calculating gravitational interactions is applied in order to determine optimal trajectories. A number of problems, necessary for determination of optimal parameters at a launch of a spacecraft and during its flyby near celestial bodies, are considered. The spacecraft trajectory was determined by numerical integration of the equations of passive motion of the spacecraft and of the equations of motion for planets, the Sun, and the Moon. The optimal trajectory of the spacecraft approaching the Sun is determined by fitting its initial conditions.

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1. INTRODUCTION

There exist space flight problems in which it is required to launch a spacecraft into a preset orbit around the Sun. For example, when a hazardous asteroid appears, multiple flights to it may become necessary for reconnaissance and investigation of its physical properties, and for taking some measures and executing technological operations in order to prevent the hazard [1]. In this connection, minimization of expenditures for these flights is of great importance.

For studying the Sun and monitoring its influence upon the Earth various orbits of spacecraft are considered. To make their launch cheaper, gravitation of the planets neighboring the Earth is used. For example, in [2] a spacecraft was considered reaching a distance of 0.18 AU (astronomical units) from the Sun in 5.8 years with the use of five gravitational maneuvers near the Earth and Venus. In other variants [3] with multiple gravitational maneuvers a spacecraft approached the Sun to distances of 0.137 AU and 0.14 AU in 1.7 and 2.5 years, respectively. In these variants the sustainer rocket engines were used for correction of trajectories.

The problem of optimal flight depends on many factors whose role can be different in each particular case. Some of them are obvious, while others can be revealed only as a result of solving particular problems. In this paper we consider the problem of optimization for a spacecraft flight to the Sun. Some of the methods developed for this task can be used for other problems of space exploration.

A possibility is considered to inject a spacecraft into an orbit in the vicinity of the Sun, provided that the trajectory is corrected using the attraction of planets rather than cruise engines. In this case one can impart the necessary initial velocity of flight to the spacecraft by booster engines near the Earth. Further on, the entire flight will proceed in the passive regime. Since no cruise engines are required for correction, the launching mass of the whole system decreases, and the costs of launching become substantially lower.

In order to realize such a variant of the flight, it is necessary to have a reliable method of calculating the flight trajectory. Only in the case when a realized trajectory coincides with that calculated in advance, one can abandon the cruise engines. Such a method of calculation of motion of the bodies under the action of gravity forces was developed by us in order to study evolution of the Solar System [4]. In addition, the motion of the spacecraft proceeds along a complicated three-dimensional curve, and it interacts with bodies at various relative velocities both during the start and in the process of motion. In order to achieve the desired results, one needs to take into account all geometrical factors of motion without exception. Since the solutions to a number of problems will be required for choosing the initial conditions and analyzing the results, we first consider them.

2. DEPENDENCE OF APPROACHING THE SUN ON THE LAUNCHING VELOCITY

A spacecraft launched from the Earth has its orbital velocity v_E. In order that the spacecraft's trajectory should be directed to the Sun, one needs to reduce this velocity, i.e., it should be launched oppositely to the Earth's orbital motion. If the spacecraft velocity relative to the Earth $v_{roc} = -v_E$, it moves to the Sun along the radius. At smaller (in magnitude) velocities of the spacecraft, its trajectory is elliptic, and in perihelion it will reach the smallest heliocentric distance R_{pr}. Let us find this distance as a function of the spacecraft's initial velocity v_{roc} based on a solution to the problem of interaction of two bodies [4, 5]. We consider the motion of

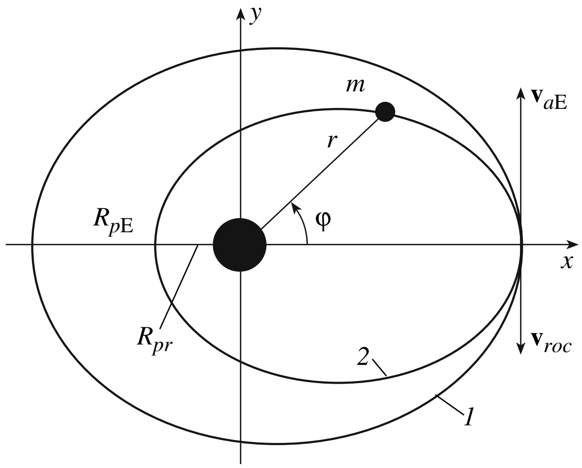

Fig. 1. The scheme of launching a spacecraft in the orbit plane of the Earth against its orbital velocity $\mathbf{v}_{aE}$.

a spacecraft and the Earth under the action of the Sun upon them. One can write the equation of a trajectory of a body with mass m, in the polar coordinate system (see Fig. 1) at whose origin a body with mass M is located, in the following dimensionless form

$$\bar{r} = \frac{1}{\sqrt{(\bar{\upsilon}_{r0}^0)^2 + (\alpha_1^0 + 1)^2}\cos(\varphi - \varphi_0) - \alpha_1^0}, \quad (1)$$

where $\bar{r} = r/r_0$ is the dimensionless radius of the spacecraft's position relative to body M; $\alpha_1^0 = \mu_1/(r_0\upsilon_{t0}^2)$ is the trajectory parameter with respect to the trajectory initial point r_0; $\mu_1 = -G(M + m)$ is the interaction parameter; G is the gravitational constant; r_0 and φ_0 are polar coordinates of the initial point; v_{t0} and v_{r0} are the transverse and radial velocities at this point; and $\bar{v}_{r0}^0 = v_{r0}/v_{t0}$ is the dimensionless radial velocity. If one chooses as a starting point the pericenter point $r_0 = R_p$, then $v_{r0} = 0$, $\bar{\upsilon}_{r0}^0 = 0$, and $\varphi_0 = 0$. In this case, Eq. (1) is simplified as

$$\bar{r} = \frac{1}{(\alpha_1 + 1)\cos\varphi - \alpha_1}, \quad (2)$$

where φ is reckoned from R_p.

Here, the parameter of trajectory α_1^0 is designated as $\alpha_1 = \mu_1/(R_p\upsilon_p^2)$, where υ_p is the spacecraft velocity at perihelion.

Notice that in this case the radius R_a of the apocenter, velocity v_a at it, and period of revolution T are determined for a closed orbit by the following expressions

$$\begin{gathered} \bar{R}_a = R_a/R_p = -1/(2\alpha_1 + 1), \\ \bar{\upsilon}_a = \upsilon_a/\upsilon_p = 1/\bar{R}_a, \\ \bar{T} = \frac{T\upsilon_p}{R_p} = -\frac{2\alpha_1\pi}{(-2\alpha_1 - 1)^{3/2}}. \end{gathered} \quad (3)$$

In addition, we write down expressions for radial ($\bar{\upsilon}_r$) and transverse ($\bar{\upsilon}_t$) velocities

$$\begin{gathered} \bar{\upsilon}_r = \upsilon_r/\upsilon_p = \sqrt{(\alpha_1 + 1)^2 - (\alpha_1 + 1/\bar{r})^2}; \\ \bar{\upsilon}_t = \upsilon_t/\upsilon_p = 1/\bar{r}; \end{gathered} \quad (4)$$

and also for orbit eccentricity e and the ratio of trajectory parameters

$$e = -(1 + 1/\alpha_1), \quad \alpha_1 = \alpha_1^0 R_p/R_0. \quad (5)$$

Equation (2) at $\alpha_1 = -1$ represents a circle; at $-1 < \alpha_1 < -0.5$ we have an ellipse; $\alpha_1 = -0.5$, $-0.5 < \alpha_1 < 0$, and $\alpha_1 = 0$ correspond to parabola, hyperbola, and straight line, respectively.

Let a spacecraft with mass m be launched at aphelion from the Earth (see Fig. 1) with the velocity v_{roc} with respect to the Earth. Here, the algebraic value of velocity v_{roc} is negative, if the spacecraft is directed oppositely to the orbital motion of the Earth. Then, the velocity of the spacecraft at its aphelion is $v_{ar} = v_{aE} + v_{roc}$, and the radius of its aphelion $R_{ar} = R_{aE}$, where parameters with indices “E” belong to the Earth, and “roc”and “r” refer to the rocket (spacecraft). According to (2), we write down trajectory parameters for the Earth and spacecraft as

$$\alpha_{1E} = \frac{\mu_1}{R_{pE}v_{pE}^2}; \quad \alpha_{1r} = \frac{\mu_1}{R_{pr}v_{pr}^2}, \quad (6)$$

where the interaction parameter $\mu_1 \approx -GM$ is assumed to be identical for these two bodies, since their masses are small in comparison to the solar mass.

Excluding μ_1 from (6) we get

$$\alpha_{1r} = \alpha_{1E}\frac{R_{pE}v_{pE}^2}{R_{pr}v_{pr}^2}. \quad (7)$$

Now, having expressed the perihelion parameters with the use of (3) through aphelion parameters, upon substituting them into (7) we obtain the equation for α_{1r},

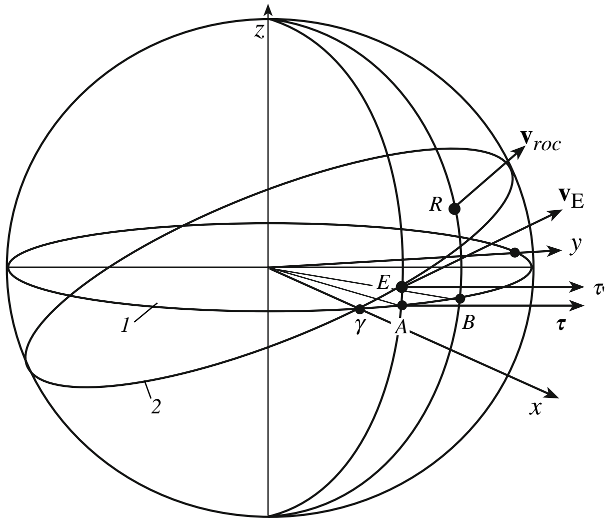

Fig. 2. Angular parameters of the Earth and spacecraft in the barycentric equatorial coordinate system: (*1*) is the celestial equator plane; (*2*) is the plane of the Earth's orbit (ecliptic); *A* and *B* are the projections of the Earth and spacecraft onto the equator circle; τ is the tangent to the equator at point *A*; $\alpha_E = \gamma A$, and $\alpha_r = \gamma B$ are the right ascensions of the Earth and spacecraft, respectively; $\delta_E = EA$ and $\delta_r = RB$ are the declinations of the Earth and spacecraft; and θ_E is the angle of inclination of the Earth's velocity vector $\mathbf{v}_E$ to the equator plane (to vector $\boldsymbol{\tau}'$).

whose solution gives us the spacecraft's trajectory parameter

$$\alpha_{1r} = \frac{\alpha_{1E}}{(1 + v_{roc}/v_{aE})^2(2\alpha_{1E} + 1) - 2\alpha_{1E}}. \quad (8)$$

Taking into account that $R_{ar} = R_{aE}$, the relative radius of spacecraft perihelion in accordance with (3) is written as (in R_{pra} the additional index *a* determines the analytical method of calculation):

$$R_{pra}/R_{aE} = -\frac{(1 + v_{roc}/v_{aE})^2(2\alpha_{1E} + 1)}{(1 + v_{roc}/v_{aE})^2(2\alpha_{1E} + 1) - 2\alpha_{1E}}. \quad (9)$$

Calculation according to formula (9) at $\alpha_{1E} = -0.9942421$ gives the following values (in astronomical units) for the nearest approach of the spacecraft to the Sun R_{pra} at its initial launching velocity v_{roc}:

v_{roc}, km/s	–10	–15	–20	–25	–30
R_{pra}, AU	0.280125	0.140905	0.058634	0.014472	$1.313 \cdot 10^{-5}$

3. OPTIMAL START FOR APPROACHING THE SUN

Under the action of all planets, the Sun, and the Moon the spacecraft motion will be distinct from that considered with the action of only the Sun. The influence of the Earth is substantial in the initial period of the flight. This will result in larger values of R_{pr} as compared to the values of R_{pra} calculated above. Afterwards, a strong influence can exert the planet near which the spacecraft trajectory will pass.

For numerical integration of the spacecraft's equations of motion jointly with 11 bodies of the Solar System the Galactica program, developed by us for studying the evolution of the Solar System, is used. The method of solution and evidence of its reliability are presented in papers [4–8]. The program allows one to solve the problems of interactions of bodies in the Solar System according to the Newton's gravity law with high precision. In [4], 9 methods of controlling the accuracy of the solution are presented, which are used for proving their reliability. The structure of errors was studied with respect to bodies, coordinates, velocities, and directions. The errors decrease with increasing orbit radius. The error in orbital direction is dominant. Therefore, the relative variation of the angular momentum δM of the entire Solar System is the most representative indicator of the accuracy. For example, when solving the problem of motion of the Solar System bodies over a period of 100 million years with a step of $\Delta T = 10^{-4}$ year and the double-length numbers (17 decimal digits), the relative error of the angular momentum of the Solar System was equal to $\delta M = 8 \cdot 10^{-11}$. Calculation with extended (up to 34 decimal digits) length of numbers and the step $\Delta T = 10^{-5}$ will give the error $\delta M = 1.5 \cdot 10^{-15}$ at computation over the 100 million years period.

The spacecraft trajectory is calculated for several years. In this period the momentum variations are equal to $\delta M \approx 10^{-15}$ or $\delta M \approx 10^{-23}$ (for extended length of numbers). The error of calculation of the spacecraft trajectory is of the same order as the error of the orbit of a celestial body with identical trajectory curvature at all points of the orbit with exception of segments of approaching another celestial body. As will be demonstrated below, the accuracy control on these segments is performed by comparing the calculated trajectory with the trajectory determined analytically in the two-body problem.

We consider the motion of a spacecraft with mass $m = 1000$ kg and radius $R_r = 5$ m, launched from the initial height $h = 300$ km from the Earth's surface in the equatorial plane. The spacecraft's location and velocity are specified (as for all other bodies) in the barycentric (with an origin at the center of mass of the Solar System) equatorial coordinate system.

When specifying the spacecraft velocity we will use for reference the Earth's velocity. According to Fig. 2

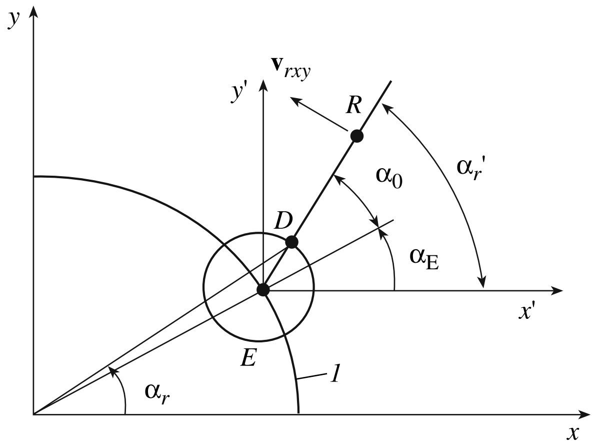

Fig. 3. The projections of positions of the Earth and spacecraft onto the barycentric equator plane: $x'Ey'$ is the geocentric equatorial coordinate system; *1* is the Earth's orbit projection onto the equator plane; $R_E = ED$ is the Earth's equatorial radius; and $H = DR$ is the initial height of the spacecraft.

the angular parameters of the Earth's locations can be written through its Cartesian coordinates x_E, y_E, z_E as

$$\alpha_E = \arcsin\frac{y_E}{\sqrt{x_E^2 + y_E^2}};\qquad \delta_E = \arcsin\frac{z_E}{\sqrt{x_E^2 + y_E^2 + z_E^2}}, \tag{10}$$

while the angular parameters of its velocity vector are equal to

$$\theta_E = \arcsin\frac{v_{zE}}{\sqrt{v_{xE}^2 + v_{yE}^2 + v_{zE}^2}};\qquad \alpha_{vE} = \arcsin\frac{v_{yE}}{\sqrt{v_{xE}^2 + v_{yE}^2}}, \tag{11}$$

where α_{vE} is the angle in the equator plane between the projection of the Earth's velocity onto this plane and axis x.

When launching a spacecraft, one can vary four angular parameters of its location and velocity vector, α_r, δ_r, θ_r, and α_{vr}. However, it is difficult to find optimal variants, because of nonlinear dependence of the final results on these parameters. Having analyzed this situation, we choose the determining parameters that control the spacecraft position in mutually perpendicular planes. These parameters are two angles, α'_r and θ_r, which lie in mutually perpendicular planes and have the values close to the angular parameters of the Earth:

$$\alpha'_r = \alpha_E + \alpha_0, \quad \theta_r = \theta_E + \theta_0 \tag{12}$$

at invariable $\delta_r = \delta_E$ and $\alpha_{vr} = \alpha_{vE}$. We emphasize that angle α'_r does not coincide with right ascension of the spacecraft, i.e., it is distinct from angle α_r in Fig. 3.

In this case, α_0 is an angle by which the spacecraft's right ascension at the initial point is ahead of the Earth's right ascension; while θ_0 is the elevation of the spacecraft velocity vector above the Earth's velocity vector with respect to the equatorial plane.

Let us project positions of the spacecraft and the Earth (see Fig. 3) onto the equator plane. In order to avoid introducing extra parameters, we direct velocity v_{rxy} perpendicular to ER. Then we can write for projections of the spacecraft location:

$$x_r = x_E + (R_E + h)\cos\alpha'_r;\qquad y_r = y_E + (R_E + h)\sin\alpha'_r;\quad z_r = z_E. \tag{13}$$

According to Fig. 2, we can write the projections of the spacecraft velocity as $v_{rz} = v_{roc}\sin\theta_r$ and $v_{rxy} = v_{roc}\cos\theta_r$. Then, according to Fig. 3, the projections of the spacecraft velocity can written in the barycentric equatorial coordinate system:

$$v_{rz} = v_{Ex} - v_{roc}\sin\alpha'_r\cos\theta_r;\qquad v_{ry} = v_{Ey} + v_{roc}\cos\alpha'\cos\theta;\qquad v_{rz} = v_{Ez} + v_{roc}\sin\theta_r. \tag{14}$$

Obtained initial conditions (13)–(14) for the spacecraft, necessary for integration of the equations of motion, are determined by three parameters: the date of launch T, advance of right ascension α_0 of the initial point, and excess over inclination θ_0 of the velocity vector in the equator plane. Figure 4 presents the results of integration of simultaneous equation of motion for the spacecraft, planets, the Moon, and the Sun when the advance angle α_0 is varied. The spacecraft is launched on November 22, 2001 (Julian date JD = 2452236.4).

Figure 4a presents the spacecraft trajectory 5 obtained by numerical integration. For the sake of comparison two analytical trajectories (6 and 7) are also presented. Hyperbolic trajectory 6 was calculated for interaction of two bodies (the Earth and spacecraft) according to formula (2). In this case, the perihelion parameters are $R_p = R_E + h$ and $v_p = v_{roc}$, the initial angle is $\varphi_0 = \alpha'_r$, and the interaction parameter is $\mu_1 = -G(m_r + m_E)$.

Analytical trajectories 6 and 7 are used in order to choose initial conditions and also to control the accuracy of integrating the equations of motion of the spacecraft at the segments when it approaches celestial bodies. We have compared calculated trajectory 5 in the frame of reference fixed to the moving Earth with hyperbolic trajectory 6 at different computation accuracies. Discrep-

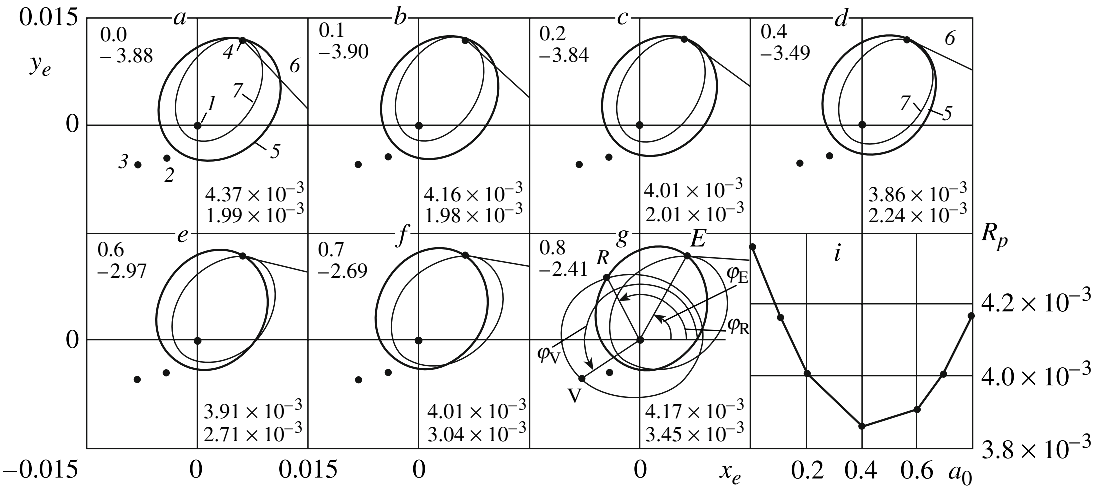

Fig. 4. Trajectory of spacecraft motion with a velocity $v_{roc} = -15$ km/s relative to the Earth for launching on November 22, 2001 at different initial advance angles α_0. All trajectories are projected onto the ecliptic plane. The bodies at the time of launch are as follows: *1* is the Sun, *2* is Mercury, *3* is Venus, *4* is the Earth, *5* is the real orbit of the spacecraft obtained by integration of the systems of equations of motion simultaneously with all bodies of the Solar System, *6* is the hyperbolic orbit with respect to the Earth ($\alpha_1 = -0.266$), and *7* is the orbit of the spacecraft on which only the Sun acts. The numbers given in each plot *a*, *b*, *c*, *d*, *e*, *f*, and *g* from top to bottom represent parameters α_0, α_1^0, R_p, and R_{pa}. Angle α is presented in radians.

ancy between the trajectories decreased with improved the accuracy of computation, and at further improvement of accuracy (for example, due to decrease of the step of computing below $\Delta T = 10^{-5}$ year) there was no change in the difference between trajectories. Therefore, all calculations were performed with this step. If necessary, the accuracy of computing can be improved by eight orders of magnitude by using the extended length of numbers.

Orbit 7 was calculated using expression (1), but for interaction of the spacecraft with the Sun. The spacecraft position at the initial time was determined relative to the Sun by the following parameters:

$$x_{rs} = x_r - x_s; \quad y_{rs} = y_r - y_s; \quad z_{rs} = z_r - z_s;$$
$$r_{rs0} = \sqrt{x_{rs}^2 + y_{rs}^2 + z_{rs}^2}, \tag{15}$$

where parameters with index *s* belong to the Sun.

Relative velocity components are defined in the similar way: v_{rsx}; v_{rsy}; v_{rsz}; and v_{rs0}. Then the angle between radius r_{rs0} and velocity v_{rs0} was determined:

$$\cos\beta = \frac{\mathbf{r}_{rs0}\mathbf{v}_{rs0}}{r_{rs0}v_{rs0}}. \tag{16}$$

Using the known angle β the transverse and radial projections of the spacecraft velocity were calculated:

$$v_{rs0t} = v_{rs0}\sin\beta; \quad v_{rs0r} = v_{rs0}\cos\beta;$$
$$\bar{v}_{r0}^0 = v_{rs0r}/v_{rs0t}. \tag{17}$$

Then the trajectory parameter $\alpha_1^0 = \mu_1/(r_{rs0}\upsilon_{rsot}^2)$ was determined, where $\mu_1 = -G(M_S + m_r)$, and the initial angle $\varphi_0 = \dfrac{\pi}{2} - \arcsin\dfrac{1+\alpha_1^0}{\sqrt{(\bar{v}_{r0}^0)^2 + (1+\alpha_1^0)^2}} + \alpha_E$. Using parameters α_1^0, $\bar{\upsilon}_{r0}^0$, and φ_0, trajectory 7 in Fig. 4a was calculated according to formula (1). The values of trajectory parameters α_1 and α_1^0, initial position angle α_0, and radii of trajectory pericenters (R_p for trajectory 5 and R_{pa} for trajectory 7) are presented in the plots. All geometrical dimensions are reduced to the characteristic size of the Solar System, $A_m = 1.09796077030958 \cdot 10^{13}$ m.

One can see in Fig. 4a that at the same initial velocity of the spacecraft $v_{roc} = -15$ km/s trajectory 7 approaches the Sun almost twice closely than the real trajectory: $R_{pa} = 1.987 \cdot 10^{-3}$ at $R_p = 4.365 \cdot 10^{-3}$. We call trajectory 5, obtained as a result of integration of equations, real, since multiple tests have shown: calculated motions of planets, the Sun, and the Moon coincide with those observed [4, 7, 8].

The reason of more distant flyby near the Sun in comparison with solution 7 of the two-body problem is caused by two circumstances. On the one hand, the spacecraft's kinetic energy is consumed to overcome the Earth's gravity attraction. Therefore, not full velocity $v_{roc} = -15$ km/s is used to reduce the spacecraft orbital velocity with respect to the Sun. On the other hand, in the process of interaction with Earth the trajectory of the spacecraft is curved so that the vector of its velocity is subtracted from the Earth's motion ineffectively. Therefore, the spacecraft trajectories in Fig. 4 are considered at variation of the position angle α_0. One can see in Fig. 4i that the spacecraft approaches the Sun

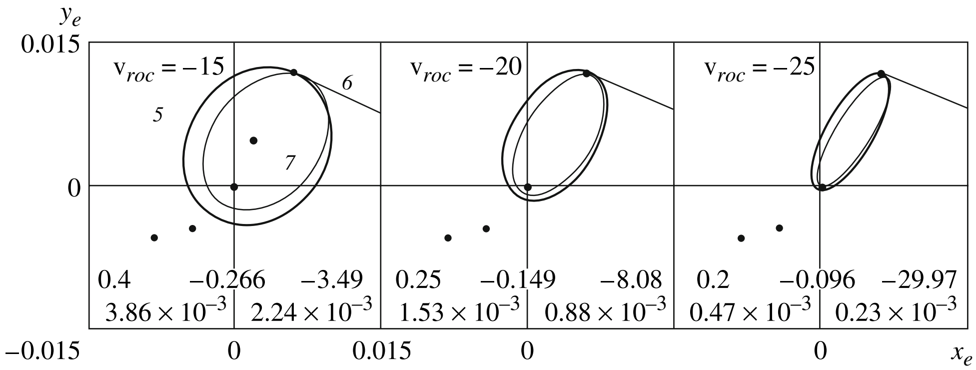

Fig. 5. The trajectories optimal in advance angle α_{0opt} at various initial velocities v_{roc}. The numbers from left to right and from top to bottom present parameters α_{0opt}, α_1, α_1^0, R_p, and R_{pa}. The remaining designations are the same as in Fig. 4.

most closely at $\alpha_{0opt} = 0.4$. In this case (see Fig. 4d) the spacecraft moving along trajectory 5 appears at the least distance from the Sun $R_p = 2.243 \cdot 10^{-3}$.In addition, hyperbolic trajectory 6 is seen to touch trajectory 5 at the initial point, i.e., in this case, vector v_{roc} of the spacecraft's relative velocity is effectively subtracted from vector v_E of the Earth's orbital motion.

Influence of angle α_0 was also studied at differing velocities, and optimal position angles α_{0opt} were established. Figure 5 presents the spacecraft orbits for optimal cases at three different launching velocities. With increased velocity v_{roc} the spacecraft approached the Sun closer. Trajectory 7 (the three-body problem) comes to the Sun almost twice nearer. Let us also note that at optimal angles α_{0opt} hyperbolic trajectories 6 become tangent to trajectory 5 at other velocities v_{roc} of the spacecraft too, i.e., optimal cancellation of the Earth's orbital velocity takes place in this case. Apparently, when the hyperbolic trajectory is tangent to the real one, the orbital velocity of the Earth will be also used in the optimal way, if the spacecraft is launched in its direction. Therefore, using this property one can facilitate the problem of finding optimal trajectories.

Table 1. Deflection angles φ_b of a spacecraft near planets (*1–9*), the Moon (*10*), and the Sun (*11*) at various spacecraft velocities at infinity v_∞

Body number	Deflection angles φ_b in radians at velocity v_∞ in km/s				
	10	15	20	25	30
1	0.16687	0.0777	0.04445	0.02868	0.02
2	0.71367	0.38768	0.23721	0.15836	0.11264
3	0.78958	0.43828	0.2711	0.18207	0.12996
4	0.22441	0.10618	0.06113	0.03955	0.02763
5	2.48497	2.18311	1.90843	1.66402	1.45019
6	2.08196	1.65622	1.31571	1.05125	0.84827
7	1.53235	1.05012	0.73903	0.53784	0.40437
8	1.65055	1.16899	0.84303	0.62425	0.47512
9	0.79887	0.44461	0.27539	0.18509	0.13218
10	0.05489	0.02477	0.01401	0.00899	0.00625
11	3.07682	3.04447	3.01214	2.97986	2.94763

4. OPTIMAL GRAVITATIONAL MANEUVER FOR APPROACHING THE SUN

In order to use attraction of celestial bodies for correction of the spacecraft trajectory, we consider the limiting angles of deflection φ_b that can be imparted by a celestial body. For a spacecraft having at infinity ($r \longrightarrow \infty$) velocity v_∞, we determine from expressions (2) and (4), respectively, the angular position φ_a (reckoned from perihelion) of the hyperbolic trajectory asymptote and the expression for the velocity:

$$\varphi_a = \arccos[\alpha_1/(\alpha_1 + 1)]; \quad v_\infty = v_p\sqrt{2\alpha_1 + 1}. \quad (18)$$

Then the angle of deflection of the velocity vector of a spacecraft during its flyby near a celestial body is written as

$$\varphi_b = 2\varphi_a - \pi = 2\arccos[\alpha_{1\infty}/(\alpha_{1\infty} + 1)] - \pi, \quad (19)$$

where the parameter of trajectory α_1, which is defined below and depends on v_∞, is designated as $\alpha_{1\infty}$. The trajectory parameter $\alpha_1 = \mu_1/(R_p \upsilon_p^2)$ depends on the pericenter velocity (v_p) and radius (R_p). We consider the spacecraft flyby at the least distance from the center of the body, equal to its radius R_b, i.e., $R_p = R_b$. The velocity v_p at the perihelion we express through the infinity velocity v_∞ according to (18). Then, the trajectory parameter looks like $\alpha_{1\infty} = \mu_1(2\alpha_{1\infty} + 1)/(R_b \upsilon_\infty^2)$. After this transformation we get the parameter of the trajectory along which the spacecraft, whose velocity at infinity is v_∞, will move:

$$\alpha_{1\infty} = \mu_1/(R_b \upsilon_\infty^2 - 2\mu_1). \quad (20)$$

When the trajectory parameter $\alpha_{1\infty}$ is known, one can determine deflection angles of the spacecraft

Table 2. Parameters of solutions with the use of a gravitational maneuver near Venus: R_{rV} is the least distance between the spacecraft and Venus; R_{bV} is the radius of Venus; t_{min} is the time of flight to the Sun; and T_{orb} is the period of circumsolar orbit

v_{roc}, km/s	α_0	θ_0	α_1	α_1^0	R_{rV}/R_{bV}	R_{pr}, AU	R_{pra}, AU	t_{min}, year	T_{orb}, year
–11.5	0.46	–0.2199	–0.4518	–2.143	6.86	0.545	0.296	0.427	0.545
–12	0.51382	–0.3296	–0.4149	–2.098	3.14	0.369	0.303	0.36	0.44
–15	0.18736827	–0.1583	–0.2655	–3.538	22.4	0.172	0.162	0.25	0.35

according to (19). It is seen in Table 1 that the largest and the smallest deflection angle φ_b for a spacecraft is produced by the Sun and the Moon, respectively. With increasing velocity the deflection angle decreases. In order that the spacecraft could escape the Earth and move away to infinity, its velocity should exceed the parabolic velocity $v_{2c} = (2\mu_1/R_b)^{0.5} = 11.18$ km/s. According to Table 1, at this velocity only Venus out of three bodies (the Moon, Venus, and Mercury) can give an appreciable deflection during a flight to the Sun. Therefore, below we consider the trajectory of a flight to the Sun executed by a spacecraft with a gravitational maneuver near Venus.

Let us determine the launch time at which the spacecraft, upon reaching the orbit of Venus, will turn out to have a rendezvous with it. Since the real trajectories of bodies are not described by known functions, and they will change at a new launch, we solve this problem by the method of successive approximations. We take advantage of a spacecraft launch on November 22, 2001 (see Fig. 4g). Let us determine the angular positions of the Earth (φ_E) and Venus (φ_V) at the launch time T_0, and of the spacecraft (φ_r) at the moment $T_0 + t_r$ of crossing the Venus orbit, where t_r is the time of the spacecraft motion on the segment ER. The angles are calculated using the coordinates of bodies in the ecliptic plane x_eSy_e with the origin of coordinates at the center of the Sun, for example, for the Earth $\varphi_E = \arctan(y_{Ee}/x_{Ee})$. In the motion time of the spacecraft its position relative to the Earth is displaced by the value $\varphi_{rE} = \varphi_r - \varphi_E$. If at the launch time T_0 Venus was ahead of the Earth by angle $\varphi_{VE} = \varphi_V - \varphi_E$, for the new launch time T_1 Venus should be behind the Earth by angle $\Delta\varphi_{VE}$ in order that a rendezvous with the spacecraft take place. Then, the time of a new launch should be displaced by the correction time

$$t_c = (\varphi_{VE} + \Delta\varphi_{VE})/(\omega_V - \omega_E), \quad (21)$$

where $\omega_V - \omega_E$ is the difference of mean angular orbital velocities of Venus and the Earth; $\Delta\varphi_{VE} = (\omega_V - \omega_E)\, t_r$.

Formula (21) is approximate, since motion characteristics are different at all points of the trajectories of bodies and at different parameters of spacecraft launch. It is shown by numerical experiments that in some cases the best result is achieved when the angular lag of Venus is $\Delta\varphi_{VE} = \varphi_{rE}$.

Taking (21) into account, the new launch time is obtained $T_1 = T_0 - t_c = 0.5105$ in centuries from December 30, 1949, which corresponds to January 20, 2001 (JD = 2451929.07). Since the period of revolution of Venus with respect to the Earth is equal to $2\pi/(\omega_V - \omega_E) =$ 1.599 years, such launches of the spacecraft for rendezvous with Venus can be repeated with this periodicity.

In order to approach Venus at a preset spacecraft velocity v_{roc}, one can vary three parameters: launch time T_1, angle of advance α_0, and angle of elevation θ_0. Calculations for the process of approaching Venus to the least distance R_{rV} were performed for several initial velocities v_{roc} and at one and the same time of launch, T_1. For every velocity, the initial angles of launch α_0 and θ_0 were calculated by the method of successive approximations with the use of fitting functions. Final values of the angles and parameters of obtained trajectories are presented in Table 2.

Figure 6 demonstrates the spacecraft trajectories for three values of starting velocities. One can see from these plots that the radius of the spacecraft orbit decreases after the interaction with Venus, and the aphelion of the spacecraft orbit does not reach the Earth's orbit. Thus, a part of the spacecraft's kinetic energy is taken away by Venus. When the initial velocity increases, starting from $v_{roc} = -11.5$ km/s, the perihelion radius decreases, and at $v_{roc} = -15$ km/s the spacecraft approaches the Sun to the distance $R_p = 0.17$ AU. After the start, it reaches this position in 0.25 years, its period of revolution being equal to 0.35 years. From a comparison with the spacecraft launch without the influence of Venus it is clear (see Fig. 4d) that the Venus influence has allowed us to reduce the orbit perihelion from 0.283 AU down to 0.17 AU. The values $R_p = 0.17$ AU is close to the distance 0.164 AU which would be reached by the spacecraft, if it were not required to overcome the Earth's attraction. Therefore, in this particular case the influence of Venus upon the spacecraft compensated its deceleration by the Earth to a considerable degree.

Thus, the gravitational maneuver near Venus resulted in a decrease of the perihelion by a factor of 0.283/0.17 = 1.7. The results presented in Fig. 5 show that at $v_{roc} = -15$, –20, and –25 km/s, respectively, $R_p =$ 0.283, 0.112, and 0.0345 AU. This allows us to determine that without gravitational maneuver near Venus one can reach $R_p = 0.17$ AU at the starting velocity $v_{roc} = -18.2$ km/s. Thus, the use of a gravitational maneuver near Venus will allow one to reduce the weight at launch and the propellant consumption for

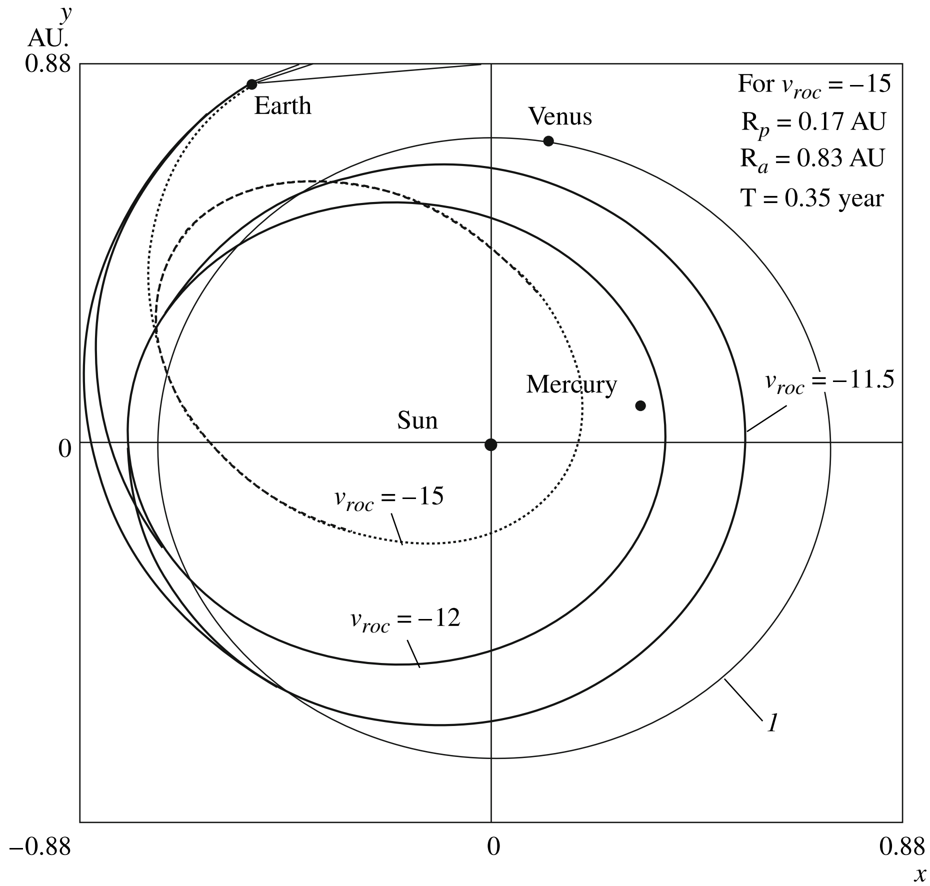

Fig. 6. Trajectories and orbits of the spacecraft launched on January 20, 2001 with different initial velocities v_{roc}. The flight is passive. After the action of Venus (on the segment crossing its orbit) the spacecraft goes into an elliptical orbit. *1* is the orbit of Venus.

imparting to the spacecraft the additional velocity $\Delta v_{roc} = -18.2 - (-15) = -3.2$ km/s. At lower velocities of launch the gravitational maneuver near Venus will be more efficient. One can see in Table 2 that in the cases considered the spacecraft passes Venus at a distance equal to several Venus's radii. Therefore, the effect of gravitational maneuvers can be increased due to a closer passage near Venus.

The orbits presented in Fig. 6 can be used for injection of spacecraft without correction cruise engines. The initial velocity $|v_{roc}|$ of the spacecraft can be only slightly higher than the escape velocity $v_{2c} = 11.18$ km/s. The time of reaching perihelion is substantially less that in other flight schemes [2, 3]. The small period of circumsolar orbit will allow the spacecraft to perform almost continuous studies of the Sun.

CONCLUSIONS

1. Solutions to a number of problems are presented in order to get analytical functions necessary for optimization of a spacecraft flight at the moments of launching from the Earth and approaching a celestial body.

2. The determining initial parameters of a spacecraft are selected to search for an optimal trajectory.

3. It is established that for optimal use of the Earth's orbital velocity the hyperbolic trajectory of the spacecraft with respect to the Earth should be tangent to its elliptic orbit relative to the Sun.

4. The use of the attraction of Venus allows one to approach the Sun closer by a factor of 1.7 at one and the same initial velocity of the spacecraft and at an identical approach to reduce the initial velocity from –18.2 km/s down to –15 km/s.

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